# Dynamics for International Commercial Fishing

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**Abstract** Since the seminal work of Smith [1969] on commercial fishing as distinct form recreational fishing, some economists have conducted dynamic analysis of commercial fishing, taking into account the biological growth law of the fish stock. Leung and Wang [1976] and Wang and Cheng [1978] have analyzed commercial fishing of a single species; Clark [1976], Solow[1976], May et al. [1979], Okuguchi [1984], Strobele and Wacker [1995], and Fischer and Mirman [1996] have investigated a more complex case of prey-predator interaction of multispecies. With the exception of Leung and Wang, and Wang and Cheng, in all these works imperfect competition has been ruled out in the market for the harvested fish; thus the price of the fish has been taken to be constant. Besides, international trade for the harvested fish has never been considered in all of the above contributions. In this paper we will formulate and analyze international commercial fishing under imperfect competition, where two countries are assumed to harvest fish of a single species in an open-access sea. The fish each country harvests is assumed to be sold not only in its own country but also in the other country. Each country's harvesting cost is assumed to be proportional to the square of its harvest and inversely proportional to the total level of the fish stock, whose intertemporal movement in the absence of fishing is assumed to be governed by the biological growth law. Our main concerns will be the existence of steady state fish stock and the long-run dynamics for the change in the fish stock, which will be classified into four possible cases, depending on the relationship between the line for $f(X)$ and the curve for $g(X)$ defined in the text.

## 1. INTRODUCTION

Since the seminal work of Smith [1969] on commercial fishing, many economists have conducted dynamic analysis of commercial fishing, taking into account the biological growth law of the fish stock. Leung and Wang [1976] and Wang and Cheng [1978] have analyzed commercial fishing of a single species; Clark [1976], Solow [1976], May et al. [1979], Okuguchi [1984] and most recently Strobele and Wacker [1995] have dealt with a more complex situation allowing for prey-predator interaction among multispecies. With the exception of Leung and Wang, and Wang and Cheng, in all these works imperfect competition has been ruled out in the market for the harvested fish; thus the price of the fish has been taken to be constant. Besides, international trade for the harvested fish has never been considered in all of the above contributions. However, international trade of identical or similar products is of paramount importance in the recent theory of international trade under imperfect competition since the seminal work of Brander [1981].

In this paper we will formulate and analyze international commercial fishing under imperfect competition, where two countries are assumed to harvest fish of a single species in an open-access sea. The fish each country harvests is assumed to be sold there as well as in the other country. Each country's harvesting cost is assumed to be proportional to the square of its harvesting rate and inversely proportional to the total level of the fish stock, whose intertemporal movement in the absence of fishing is assumed to be governed by the biological growth law originally formulated by Verhulst. We will find that the long-run dynamics of the change in the fish stock will be classified into four possible cases, depending on the relationship between the line for $f(X)$ and the curve for $g(X)$ defined in the next section, where $Xf(X)$ is the rate of change of the fish stock when there is no harvesting and $Xg(X)$ is the harvesting rate of the fish.

## 2. INTERNATIONAL DUOPOLY MODEL

Suppose two countries engaged in commercial fishing in an open-access sea and in selling the harvested fish in the two countries, the home and foreign countries. If $X$ is the fish stock, its rate of change in the absence of fishing is assumed to be governed by the biological growth law,

$$dX/dt = X(\alpha - \beta X), \qquad (1)$$

where $\alpha$ and $\beta$ are positive constants. If the fish stock is small, there will be abundant foods for the fish, and its stock will grow at the intrinsic growth rate $\alpha$. As the fish stock becomes larger, there will be competition among the fish for the limited foods, forcing the growth rate to decrease. Rewriting (1), we have

$$dX/dt = \alpha X(1 - KX), \quad K \equiv \beta/\alpha. \qquad (1')$$

This is the logistic equation originally due to P.F.Verhulst. The constants $\alpha$ and $1/K$ are the intrinsic growth rate and carrying capacity, respectively. If fishing is absent, the fish stock converges to $K$ in the long-run.

Let $x_{ij}$ be the amount fish harvested by country $i$ and sold in country $j$, $i,j=1,2$. The inverse demand functions for fish in the two countries are assumed to be linear and given by

$$p_1 = a_1 - b_1 (x_{11} + x_{21}), \qquad (2.1)$$

$$p_2 = a_2 - b_2 (x_{12} + x_{22}), \qquad (2.2)$$

where $a_i$ and $b_i$ are positive constants for $i$=1,2, $p_1$ and $p_2$ are the prices of the fish in country 1 and country 2, respectively. Given the fish stock which is not evenly distributed in a open-access sea, the unit cost of harvesting the fish will increase as the harvesting rate increases and will be proportional to it. Hence under the same condition, the total harvesting cost will be proportional to the square of the harvesting rate. If, on the other hand, the harvesting rate is given, the harvesting will be easier and less costly as the fish stock increases. Hence given the harvesting rate, the harvesting cost is inversely proportional to the fish stock. We assume therefore that each country's harvesting cost is proportional to the square of its harvesting rate and inversely proportional to the fish stock. If $c_i$ is country $i$'s opportunity cost for fishing, country 1' and 2's profits, $\pi_1$ and $\pi_2$, are given by

$$\pi_1 = p_1 x_{11} + p_2 x_{12} - \gamma_1 ( x_{11} + x_{12})^2 / X - c_1, \qquad (3.1)$$

$$\pi_2 = p_1 x_{21} + p_2 x_{22} - \gamma_2 ( x_{21} + x_{22})^2 / X - c_2, \qquad (3.2)$$

where $\gamma_i$ and $c_i$ are positive constants, $i$=1,2. We assume that two countries behave as Cournot duopolists.

Let $X_i$ and $Y_i$ be the total harvest of and the total supply to country $i$, respectively.

$$X_i \equiv x_{i1} + x_{i2}, \quad i=1,2.$$

$$Y_i \equiv x_{1i} + x_{2i}, \quad i=1,2.$$

Given $X$, the first order conditions for country 1's and country 2's profit maximization are given by (4.1) and (4.2), and (4.3) and (4.4), respectively, where a corner maximum is assumed to be ruled out.

$$x_{11} = a_1 / b_1 - Y_1 - 2\gamma_1 X_1 / b_1 X. \qquad (4.1)$$

$$x_{12} = a_2 / b_2 - Y_2 - 2\gamma_1 X_1 / b_2 X. \qquad (4.2)$$

$$x_{21} = a_1 / b_1 - Y_1 - 2\gamma_2 X_2 / b_1 X. \qquad (4.3)$$

$$x_{22} = a_2 / b_2 - Y_2 - 2\gamma_2 X_2 / b_2 X. \qquad (4.4)$$

Before proceeding further, let

$$A \equiv a_1/b_1 + a_2/b_2.$$

$$B \equiv 1/b_1 + 1/b_2.$$

$$S \equiv Y_1 + Y_2 = X_1 + X_2.$$

Adding (4.1) and (4.2),

$$X_1 = A - S - 2B\gamma_1 X_1 / X \qquad (5.1)$$

Similarly from (4.3) and (4.4),

$$X_2 = A - S - 2B\gamma_2 X_2 / X. \qquad (5.2)$$

Hence (5.1) and (5.2) yield (6.1) and (6.2), respectively.

$$X_1 = (A-S) / (1+2B\gamma_1 / X), \qquad (6.1)$$

$$X_2 = (A-S) / (1+2B\gamma_2 / X). \qquad (6.2)$$

Adding (6.1) and (6.2), taking account into account $X_1 + X_2 = S$, and solving with respect to $S$, we have

$$S \equiv S(X) = \{X(X+2B\gamma_1)+X(X+2B\gamma_2)\}A / \{(X+2B\gamma_1)(X+2B\gamma_2) + X(X+2B\gamma_1) + X(X+2B\gamma_2)\} = 2X\{X+B(\gamma_1+\gamma_2)\}A / \{3X^2 + 4B(\gamma_1+\gamma_2)X + 4B^2\gamma_1\gamma_2\} \equiv Xg(X), \qquad (7)$$

which gives the total harvest by two countries as a function of the fish stock.

Hence, the differential equation governing the change of the fish stock in the presence of international commercial fishing is

$$dX / dt = X(f(X) - g(X)), \qquad (8)$$

where

$$f(X) = \alpha - \beta X, \qquad (9)$$

$$g(X) \equiv S(X) / X, \qquad (10)$$

where $S(X)$ is given by (7).

The nonextinct steady state or the bionomic equilibrium is characterized as the solution of the following equation.

$$f(X) = g(X). \qquad (11)$$

We have to examine the properties of the curve for $g(X)$ in order to analyze dynamics for the fish stock movement. Noting that the qualitative properties of $g(X)$ and

$$h(X) \equiv g(X) / 2A \qquad (12)$$

are identical, we get from

$$h'(X) = -\{3X^2+6B(\gamma_1+\gamma_2)X + 4B^2(\gamma_1^2+\gamma_2^2+\gamma_1\gamma_2)\} / \{3X^2+4B(\gamma_1+\gamma_2)X+4B^2\gamma_1\gamma_2\}^2 < 0 \quad \text{for all} \quad X \geq 0, \qquad (13.1)$$

and

$$h''(X) = [18X^3 + 54B(\gamma_1+\gamma_2)X^2 + 72B^2(\gamma_1^2+\gamma_2^2+\gamma_1\gamma_2)X + 8B^3(\gamma_1+\gamma_2)\{(\gamma_1-\gamma_2)^2+3(\gamma_1^2+\gamma_2^2)\}] / 3X^3 + 4B(\gamma_1+\gamma_2) + 4B^2\gamma_1\gamma_2\}^3 > 0 \quad \text{for all} \quad X \geq 0, \qquad (13.2)$$

respectively,

$$g'(X) < 0 \quad \text{for all} \quad X \geq 0, \tag{14.1}$$

and

$$g''(X) > 0 \quad \text{for all} \quad X \geq 0. \tag{14.2}$$

Hence, $g(X)$ is strongly decreasing, convex function of $X$.

We are now in a position to analyze the existence and stability of the nonextinct steady state fish stock. Four possibilities emerge:

*Case 1.* Equation (11) has no real solution.

*Case 2.* It has only one real solution and $g(0) > \alpha$.

*Case 3.* It has only one real solution and $g(0) < \alpha$.

*Case 4.* It has two distinct real solutions.

These four cases are illustrated in Figures 1-4.

First, consider Figure 1. In this case the harvest rate of the fish stock is always greater than its biological growth rate, resulting in extinction of the fish stock. In Figure 2, the harvest rate is greater than the biological growth rate regardless of the initial stock level unless it equals $X^*$. Hence, the fish stock converges to the steady state level $X^*$ if the initial stock is larger than $X^*$ but becomes extinct if the initial stock is less than $X^*$. In Figure 3, the harvest rate is greater(less) than the growth rate for $X > X^*$ ( $X < X^*$ ). Hence the fish stock converges to the steady state level regardless of the initial stock level. In Figure 4, if the initial stock is larger than the larger steady state stock $X^{**}$, the fish stock decreases and converges to $X^{**}$; if the initial stock is less than the smaller steady state stock $X^*$, the fish stock decreases and become extinct in the long-run. If, however, the initial stock lies between $X^*$ and $X^{**}$, the biological growth rate is greater than the harvest rate, thus the fish stock increases and converges to the larger steady state level $X^{**}$.

## 3. INTERNATIONAL OLIGOPOLY

We can easily extend our international duopoly fishery model to international oligopoly fishery where more than two countries engage in fishing and trade the harvested fish one another. Let therefore be $n$ countries, and

$$p_i = a_i - b_i Y_i \quad , \qquad i = 1,2,\ldots,n, \tag{15}$$

be the demand function for the $i$ th country, where

$$Y_i \equiv x_{1i} + x_{2i} + \ldots + x_{ni}, \quad i = 1,2,\ldots,n,$$

is the total supply of the harvested fish to the $i$-th country. Its total harvest rate is

$$X_i \equiv x_{i1} + x_{i2} + \ldots + x_{in}, \quad i = 1,2,\ldots,n,$$

and the cost function is given by

$$C_i = c_i + \gamma_i X_i^2 / X \quad . \qquad i = 1,2,\ldots,n. \tag{16}$$

By definition the $i$-th country's profit function is

$$\pi_i \equiv \sum_j p_j x_{ij} - c_i - \gamma_i X_i^2 / X \quad , \quad i, j = 1,2,\ldots,n. \tag{17}$$

Assume that all countries behave as Cournot oiogopolists. Then deriving and arranging the first order conditions for profit maximization, we finally get

$$S(X) = Aw(X) / (1 + w(X)), \tag{18}$$

$$w(X) \equiv \sum_j X / (X + 2\gamma_j B) \tag{19}$$

and

$$A \equiv \sum_j a_j / b_j, \qquad B \equiv \sum_j 1 / b_j.$$

We let as in Section 2,

$$g(X) \equiv S(X) / X. \tag{20}$$

After a lot of calculations, we get

$$g'(X) < 0, \quad g''(X) > 0 \quad \text{for all} \quad X \geq 0. \tag{21}$$

Thus four possibilities arise for the existence and stability of the nonextinct steady state fish stock as in Section 2. See Szidarovszky and Okuguchi [1996] for the details.

## 4. CONCLUDING REMARKS

In Section 2 of this paper we have formulated international duopoly model of fishery under imperfect competition, where two countries are assumed to harvest the fish of a single species from an open-access sea and to sell the harvested fish in both countries. We have shown that four cases are conceivable regarding the existence and stability or instability of the nonextinct steady state level of fish stock. In one case there exists a unique nonextinct steady state fish stock which is globally stable. In Section 3 we have briefly indicated how our analysis in Section 2 can be extended to deal with international oligopoly in fishery where more than two countries engage in harvesting activities. Though mathematical analysis for oligopoly becomes more complicated than for duopoly, no qualitative difference has been observed regarding dynamics for duopolistic and oligopolistic international fisheries.

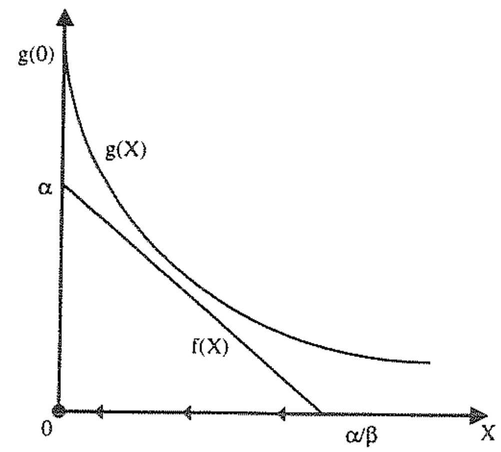


Figure 1. Case 1.

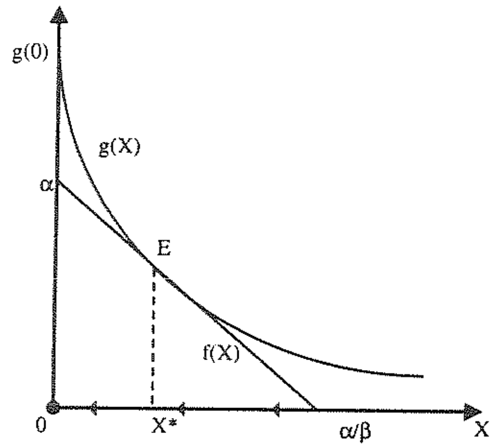


Figure 2. Case 2.

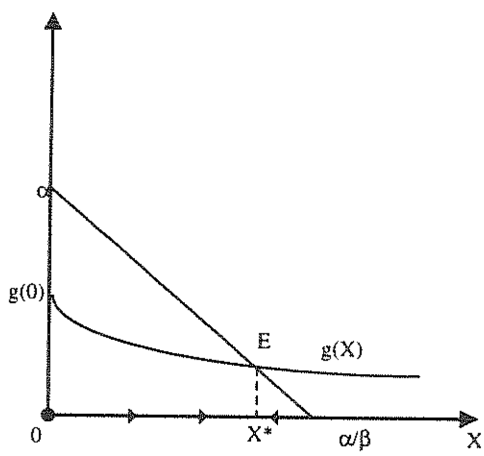


Figure 3. Case 3.

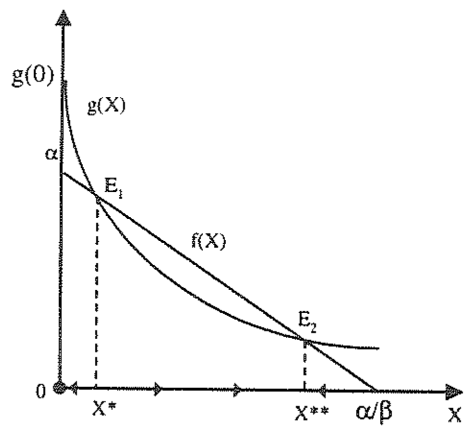


Figure 4. Case 4.

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